A METHOD OF ESTIMATING THE RISK OF A CATASTROPHIC FAILURE TO ENGINE'S ROTATING MEMBERS AS RELATED TO FATIGUE – AN OUTLINE

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Abstract

The paper has been intended to introduce a probabilistic method to estimate – from the standpoint of fatigue – the risk of a catastrophic failure to rotating (moving) members of an aircraft engine, i.e. to compressor blades. It has been assumed that there is a hidden defect in the material's structure, which initiates a small-size crack. Load-affected, the crack keeps growing. The crack propagation dynamics, when approached in a deterministic way, remains consistent with the Paris formula. The crack growth is effected by some random load characterised with the servicing load spectrum. While determining the load spectrum, all possible operational events are taken into account, excluding ones that could result in an immediate damage to the component. It has been assumed that random instances of load increase, which may result in an immediate damage, compose a separate set of events; hence, they have not been taken into account in this model.

A partial differential equation of the Fokker-Planck type has been used to describe randomly approached dynamics of crack propagation. Having solved this equation enables a density function of the fatigue crack length to be found. This function, in turn, has been used to determine the risk of a catastrophic failure to a compressor blade. Furthermore, this function can also be used to find safe service life of the structure under consideration.

***Keywords**: risk of failure, fatigue, load spectrum, probability density, stress intensity factor*

1. Introduction

Maximum utilisation of aircraft (the fleet) on the one hand, and on the other hand, providing some reasonable levels of reliability and flight safety both make the aero-engine's health monitoring a real demand. Numerous papers on these issues have already been published [1, 2, 3, 4].

Many factors affect reliable engine operation. Airflow ingested into the engine inlet is one of them. Parameters of the airflow passing through the compressor are featured with considerable circumferential and radial irregularities as well as time variability. It results, first and foremost, from external, engine-operation affecting agents, such as: changes in flight conditions, atmospheric turbulence, backwash (i.e. following in wake of another aircraft in formation flight), air inlet icing, or even swallowing and deposition of foreign matter. Hence, the irregularity and instability of the airflow onto the blades results in blade twist and vibration, and consequently, in fatigue crack growth.

Under some specific compressor operating conditions, airflow disturbance may occur, resulting in compressor surging. Any instance of this is dangerous, since resonant vibration of high amplitude is initiated. Even more dangerous are: ingestion of birds (or other objects of this size) and deposition thereof upon inlet guide vanes (e.g. in an engine with subsonic-flow compressor), the icing of air inlet or inlet guide vanes. All these may result in very large airflow disturbances inducing blade vibration of a very large amplitude [1]. Hence, stresses measured at the blade root can increase twofold during the compressor surging; deposition of any foreign matter can effect stress increase even up to the value approximating fatigue strength of blades. In a very short time it may cause, in turn, some damage to the engine. On the other hand, ingestion of some foreign matter of sufficiently high mass can result in the immediate engine damage.

Operational practice and experience prove that the following engine components most often suffer damages/failures:

- compressor blades (most often, first-stage blades),
- turbine blades,
- the bearing system,
- combustion chamber.

On the grounds of the processed statistical data on damages/failures to aero-engines one can easily find that first stages of an engine compressor are most severely exposed to damages. Hence, it has been decided to focus all the attention on estimating the risk of having these components damaged.

The above-presented description of phenomena that might occur in the course of operation indicates there are two most essential reasons for catastrophic failures to compressor blades, namely fatigue cracking and an immediate failure resulting from the foreign matter ingestion.

With the above taken into consideration, a set of possible events has been defined. Events included enable us to describe – with the following formula (eq (1)) – the probability that a catastrophic failure to the aircraft engine occurs (i.e. the risk that such a failure occurs):

$$Q(t) = (1 - Q_{CO}(t)) \cdot Q_{ZZ}(t) + Q_{CO}(t) \cdot Q_{ZN}\,, \tag{1}$$

whereas the probability that a catastrophic failure does not occur to the aircraft engine can be written down in the following way:

$$R(t) = (1 - Q_{CO}(t)) \cdot (1 - Q_{ZZ}(t)) + Q_{CO}(t) \cdot (1 - Q_{ZN})\,, \tag{2}$$

with $Q(t) + R(t) = 1$.

The checkup of the suggested dependences (1) and (2) gives:

$$(1 - Q_{CO}(t)) \cdot Q_{ZZ}(t) + Q_{CO}(t) \cdot Q_{ZN} + (1 - Q_{CO}(t)) \cdot (1 - Q_{ZZ}(t)) + Q_{CO}(t) \cdot (1 - Q_{ZN}) = 1\,,$$

where:

$Q_{CO}(t)$ - probability of the foreign matter ingestion within flight time interval $(0, t)$,

Q_{ZN} - probability of immediate engine failure,

$Q_{ZZ}(t)$ - probability of fatigue failure within flight time interval $(0, t)$.

These probabilities form separate sets of events.

If we assume that $Q_{CO}(t) = 0$, then $Q(t) = Q_{ZZ}(t)$. Hence, to find the risk of a catastrophic failure to the aircraft engine, the risk of fatigue failure to compressor blades $Q_{ZZ}(t)$ has to be found. This is the subject matter of this paper.

It should be emphasised that particular instances do not occur separately. They compose a mix of events. All the instances that result in a damage/failure to the compressor blade are conditioned by operational events. Having found a suitable form of the operational blade-loading spectrum, one can present a model that enables us to find dependences to determine the risk of a damage/failure to an engine blade. This paper is an attempt to introduce such a method. In this method it has been assumed that the load spectrum does not contain stresses of values high enough to cause an immediate failure to the structure (fatigue process is interrupted by immediate rupture).

2. Estimation of risk of catastrophic failures to engine compressor blades, from the standpoint of fatigue

This section is an attempt to introduce a method to estimate the risk that a failure occurs to a compressor blade of an aero-engine. It has been assumed that:

- fatigue load affecting a component is determined by the load spectrum throughout the engine operation (conditioned by disturbances in the airflow due to varying aircraft flight conditions),
- crack growth process, approached in the deterministic way, is described with the Paris formula, for $m \neq 2$, in the following way:

$$\frac{da}{dN_z} = C M_k^m \, E[(\sigma_{\max})^m] \pi^{\frac{m}{2}} a^{\frac{m}{2}}, \tag{3}$$

where:

C, m - material constants, with $m \neq 2$,
a - crack length,
N_z - the number of fatigue cycles,
M_k - coefficient of the component's dimensions and crack location.

The value of equivalent load (determined on the grounds of the load spectrum) is as follows:

$$E[(\sigma_{\max})^m] = P_1(\sigma_1^{\max})]^m + P_2[(\sigma_2^{\max})]^m + \ldots + P_L[(\sigma_L^{\max})]^m, \tag{4}$$

$$\sigma_i^{\max} = \frac{\sigma_i^{\max} + \sigma_i^{\min}}{2} + \sigma_i^a, \quad i = 1,2,\ldots,L, \tag{5}$$

$\sigma_i^{\max}$ - the maximum value of cyclic load in the *i*-th threshold (of discrete load value),
$\sigma_i^{\min}$ - the minimum value of cyclic load in the *i*-th threshold,
σ_i^a - amplitude of cyclic load in the *i*-th threshold,
P_i - frequency of threshold values of load, defines with the following dependences:

$$P_i = \frac{n_i}{N_c}, \qquad N_c = \sum_{i=1}^{L} n_i, \tag{6}$$

n_i - the number of repetitions of some specific threshold values of load during one flight for the aircraft engine (e.g. a standard flight),
N_c - the total number of load cycles on compressor blades in the course of aircraft standard flight (with account taken of all possible load instances).

Eq (3) could be expressed against time or, more precisely, aircraft flying time:

$$N_z = \lambda t \ , \tag{7}$$

where:

λ - intensity of occurrence (re-occurrence) of fatigue load intensity on the engine blade,

t - aircraft flying time.

In our case, $\lambda = \frac{1}{\Delta t}$, where Δt – load cycle duration. The working formula to determine Δt can be accepted in the following form:

$$\Delta t = \frac{T}{N_c} \ , \tag{8}$$

where:

T - time of aircraft standard flight.

Using the assumptions made and notation applied, one can set abort describing the dynamics of crack propagation in the member, from the probabilistic standpoint.

To do that, the following difference equation has been used in [5]:

$$U_{a,t+\Delta t} = P_1 U_{a-\Delta a_1,t} + P_2 U_{a-\Delta a_2,t} + \ldots + P_L U_{a-\Delta a_L,t} \ , \tag{9}$$

where:

$U_{a,t}$ - probability that for the flying time equal to *t*, the crack length was *a*,

Δa_i - crack growth throughout time interval Δt for stress value $\sigma_i^{\max}$,$(i = 1,2,\ldots,L)$.

The following differential equations of the Fokker-Planck type have been arrived at from eq (9) in [5]:

$$\frac{\partial u(a,t)}{\partial t} = -\alpha(a)\frac{\partial u(a,t)}{\partial a} + \frac{1}{2}\beta(a)\frac{\partial^2 u(a,t)}{\partial a^2} \ , \tag{10}$$

where:

$u(a,t)$ - density function of crack length dependent on aircraft flying time,

$\alpha(a)$ - coefficient that determines a mean crack-length increment per time unit, defined with the following dependence:

$$\alpha(a) = \lambda \sum_{i=1}^{L} P_i \Delta a_i \ , \tag{11}$$

$\beta(a)$ - the square of the crack length increment as related to a time unit, defined with the following dependence:

$$\beta(a) = \lambda \sum_{i=1}^{L} P_i (\Delta a_i)^2 \ , \tag{12}$$

Δa_i - crack length increment defined with the following dependence:

$$\Delta a_i = C_m (\sigma_i^{\max})^m a^{\frac{m}{2}} \ , \tag{13}$$

$$C_m = C M_k^m \pi^{\frac{m}{2}} \ . \tag{14}$$

With the notation applied, eq (3) can be written down in the following form:

$$\frac{da}{dt} = \lambda C M_k^m E[(\sigma_{\max})^m] \pi^{\frac{m}{2}} a^{\frac{m}{2}}. \tag{15}$$

The following dependence is a solution to eq (15)

$$a = (a_o^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda C M_k^m E[(\sigma_{\max})^m] \pi^{\frac{m}{2}} t)^{\frac{2}{2-m}}. \tag{16}$$

Using eq (14), coefficients $\alpha(a)$ and $\beta(a)$ can be expanded into the following forms:

$$\alpha(t) = \lambda C M_k^m \pi^{\frac{m}{2}} E[(\sigma^{\max})^m] \cdot [a_o^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda C M_k^m E[(\sigma_{\max})^m] \pi^{\frac{m}{2}} t]^{\frac{m}{2-m}}, \tag{17}$$

$$\beta(t) = \lambda C^2 M_k^{2m} \pi^m E[(\sigma^{\max})^{2m}] \cdot \{[a_o^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda C M_k^m E[(\sigma_{\max})^m] \pi^{\frac{m}{2}}]^{\frac{2}{2-m}}\}^m. \tag{18}$$

With eqs (17) and (18) taken into consideration, eq (10) takes the form:

$$\frac{u(a,t)}{\partial t} = -\alpha(t) \frac{\partial u(a,t)}{\partial a} + \frac{1}{2} \beta(t) \frac{\partial^2 u(a,t)}{\partial a^2}. \tag{19}$$

Solution of eq (19) is the function of crack length versus flying time that is searched for:

$$u(a,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(a-B(t))^2}{2A(t)}}, \tag{20}$$

where:

$B(t)$ - a mean for flying time t,
$A(t)$ - variance.

Computational formulae take the forms:

$$B(t) = \int_0^t \alpha(z) dz, \tag{21}$$

$$A(t) = \int_0^t \beta(z) dz. \tag{22}$$

Having computed the integrals, we arrive at the following dependences:

$$B(t) = [a_o^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda C M_k^m \pi^{\frac{m}{2}} E[(\sigma^{\max})^m] t]^{\frac{2}{2-m}} - a_o, \tag{23}$$

$$A(t) = \frac{2}{2+m} C M_k^m \pi^{\frac{m}{2}} \frac{E[(\sigma^{\max})^{2m}]}{E[(\sigma^{\max})^m]} [(a_o^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda C M_k^m \pi^{\frac{m}{2}} E[(\sigma^{\max})^m] t)^{\frac{2+m}{2-m}} - a_o^{\frac{2+m}{2}}]. \tag{24}$$

To evaluate the critical crack length, we need to use the stress intensity factor in the form:

$$K = M_k \sigma \sqrt{\pi a} \ , \tag{25}$$

where:

M_k - correlation coefficient that comprises geometric characteristics of the component's dimensions and crack location,

σ - a member-affecting load.

The with eq (25) defined factor becomes the critical quantity K_c called 'material's crack resistance' (fatigue strength), if critical crack length a_{kr} and critical stress σ_{kr} have been reached:

$$K_c = M_k \sigma_{kr} \sqrt{\pi a_{kr}} \ . \tag{26}$$

Using eq (26), the critical crack length can be evaluated:

$$a_{kr} = \frac{K_c^2}{M_k^{\,2} \sigma_{kr}^2 \, \pi} \ . \tag{27}$$

The exceeding of the critical crack length results in a catastrophic failure to the engine blade. If factor of safety is introduced, admissible value of the crack length can be found. The computational formula takes the following form:

$$a_d = \frac{K_c^2}{k M_k^{\,2} \sigma_{kr}^2 \, \pi} \ , \tag{28}$$

where:

k - factor of safety,

σ_{kr} - maximum value of operation-induced stress that affects the aircraft member.

With the crack-length density function (20) and eq (27), one can find the dependence to estimate the risk of a catastrophic failure to a compressor blade for flying time *t*:

$$Q_{ZZ}(t) = \int_{a_{kr}}^{\infty} u(a,t)da \ . \tag{29}$$

On the other hand, if account is taken of the factor of safety, the risk of a failure to the component will be estimated with the following dependence:

$$\overline{Q}_{ZZ}(t) = \int_{a_d}^{\infty} u(a,t)da \ . \tag{30}$$

3. Summary

The in the present paper found: density function of fatigue crack length, and dependences to estimate the risk of damages/failures to the engine's rotating (moving) members, i.e. compressor blades, may prove extremely helpful in aeronautical systems dedicated reliability and life-cycle studies.

Taking account of random values of stresses in rotating (moving) members of engines is a great advantage of this method, as is its applicability to materials, for which *m* in the Paris dependence is different from two (2). The in the Introduction presented dependence that describes the risk of a catastrophic failure to an engine, with account taken of possibility of the foreign matter ingestion, needs to take another possibility into consideration (for the model under discussion). It is the possibility of 'breaking' (interrupting) the crack growth, i.e. of immediate failure (to be considered separately).

To use this method for practical purposes, there is a demand and intention to verify it on the grounds of both real operating data that enable determination of load spectrum for engine's compressor blades, and indispensable material data.

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